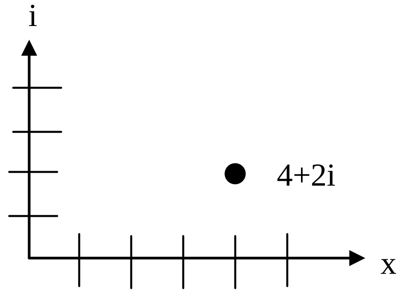
Complex Numbers

- The introduction of the number i allows us to solve equations like $x^2 + 1 = 0$, which traditionally we have stated as having no solutions (or no real solutions) as we cannot take the square root of a negative number.
- Using the definition $i^2 = -1$, i.e. $i = \sqrt{-1}$, we can solve equations like $x^2 - 2x + 5 = 0$.
- Complex numbers are in general written in the form $z = a + bi$, where $'a'$ is the real part, and $'b'$ is the imaginary part.
- Complex numbers exist on the Argand plane. On an Argand diagram, the horizontal axis is called the **real axis**, and the vertical axis is called the **imaginary axis**.



Questions

1) If $a = 3 - 2i$ and $b = -7 + 3i$, find:

a) $a + b$:

$$a + b = (3 - 2i) + (-7 + 3i) = 3 - 7 - 2i + 3i = -4 + i$$

b) $a - b$

$$a - b = (3 - 2i) - (-7 + 3i) = 3 + 7 - 2i - 3i = 10 - 5i$$

c) ab

$$\begin{aligned} ab = (3 - 2i)(-7 + 3i) &= 3 \cdot -7 + 3 \cdot 3i + -2i \cdot -7 + -2i.3i \\ &= -21 + 9i + 14i - 6i^2 \\ &= -21 + 9i + 14i - 6(-1) \\ &= -15 + 13i \end{aligned}$$

2) Expand $(2 + i)^3$ in full.

$$\begin{aligned} (2 + i)^3 &= (2 + i)(2 + i)(2 + i) = (4 + 4i + i^2)(2 + i) \\ &= (4 + 4i - 1)(2 + i) = (3 + 4i)(2 + i) = 6 + 3i + 8i + 4i^2 \\ &= 6 + 11i + 4(-1) = 2 + 11i \end{aligned}$$

Delta Ex 30.2 Q1, 4   Ex 30.3   Ex 30.4